Calculus

Thomas pg 169, 2-12all, 14,16,22,24,25

2) $s(t) = 6t - t^2$ $[0,6]$

a) $s(0) = 0$ displacement = 0 m
$s(6) = 0$

ave velocity $= \frac{0+0}{6-0} = 0$ m/sec

b) $v(t) = 6 - 2t$ $a(t) = -2$ m/s²
$v(0) = 6$ m/s
$v(6) = -6$ m/s

c) $v(t) = 6 - 2t = 0$
$6 = 2t$ at 3 sec
$t = 3$

4) $s(t) = \frac{1}{4}t^4 - t^3 + t^2$ $[0,3]$

a) $s(0) = 0$
$s(3) = 2.25$ displacement 2.25 m

ave velocity $= \frac{2.25 - 0}{3-0} = .75$ m/sec

b) $v(t) = t^3 - 3t^2 + 2t$ $a(t) = 3t^2 - 6t + 2$
$v(0) = 0$ m/s $a(0) = 2$ m/s²
$v(3) = 6$ m/s $a(3) = 11$ m/s²

c) $v(t) = t^3 - 3t^2 + 2t = 0$
$t(t-1)(t-2) = 0$
at 0 sec, 1 sec, 2 sec

3) $s(t) = -t^3 + 3t^2 - 3t$ $[0,3]$

a) $s(0) = 0$ displacement = −9 m
$s(3) = -9$

ave velocity $= \frac{-9+0}{3-0} = -3$ m/sec

b) $v(t) = -3t^2 + 6t - 3$ $a(t) = -6t + 6$
$v(0) = -3$ m/sec $a(0) = 6$ m/s²
$v(3) = -12$ m/sec $a(3) = -12$ m/s²

c) $v(t) = -3t^2 + 6t - 3 = 0$
$-3(t-1)^2 = 0$
1 sec

5) $s(t) = t^3 - 6t^2 + 9t$

a) $v(t) = 3t^2 - 12t + 9$ $a(t) = 6t - 12$
$0 = 3t^2 - 12t + 9$
$0 = 3(t-3)(t-1)$ $a(1) = -6$ m/s²
$\{1, 3\}$ $a(3) = 6$ m/s²

b) $a(t) = 6t - 12 = 0$
$6t = 12$
$t = 2$ sec

$v(2) = 3(2)^2 - 12(2) + 9$
$= -3$ m/s
speed 3 m/s

c) on (0,1) $s(0) = 0$, $s(1) = 4$, $= 4$
on (1,2) $s(1) = 4$, $s(2) = 2$, $= |-2| = 2$ total = 6

Actually, since 1 is a double root, the object changes direction twice at 1 second which results in no change of direction.

6a) $v(t) = t^2 - 4t + 3$ $a(t) = 2t - 4$
$0 = (t-3)(t-1)$ $a(1) = -2$
1 sec, 3 sec $a(3) = 2$

b) Forward $[0,1)$
back $(1,3)$
Forward $(3,\infty)$

c) decreasing on $(0,2)$
increasing on $(2,\infty)$

7)

Mars	Jupiter
$s(t) = 1.86t^2$	$s(t) = 11.44t^2$
$v(t) = 3.72t$	$v(t) = 22.88t$
$27.8 = 3.72t$	$27.8 = 22.88t$
$7.473 = t$	$1.215 = t$
7.473 sec	1.215 sec

8) $s(t) = 24t - .8t^2$

a) $v(t) = 24 - 1.6t$

$a(t) = -1.6$

b) $0 = 24 - 1.6t$

$1.6t = 24$

$t = 15$ sec

c) $s(15) = 24(15) - .8(15)^2$

$= 180$ m

d) $90 = 24t - .8t^2$

$.8t^2 - 24t + 90 = 0$

$\{4.393, 25.607\}$

4.393 sec

e) 30 sec (15×2)

9) $s(t) = 15t - (\frac{1}{2}g)t^2$

$v(t) = 15 - gt$

$0 = 15 - g(20)$

$20g = 15$

$g = .75\ m/s^2$

	Moon	Earth
10)	$s(t) = 832t - 2.6t^2$	$s(t) = 832t - 16t^2$
	$v(t) = 832 - 5.2t$	$v(t) = 832 - 32t$
	$0 = 832 - 5.2t$	$0 = 832 - 32t$
	$5.2t = 832$	$32t = 832$
	$t = 160$ sec	$t = 26$ sec
a)	320 sec ⟷	52 sec
b)	$s(160) = 832(160) - 2.6(160)^2$	$s(26) = 832(26) - 16(26)^2$
	= 66560 Ft ⟷ 12.606 mi	= 10816 Ft ⟷ 2.048 mi

11) $s(t) = 179 - 16t^2$

$v(t) = -32t$

speed $= |-32t|$

$a(t) = -32$

b) $0 = 179 - 16t^2$

$16t^2 = 179$

$t =$ 3.345 sec

c) $v(3.345) = -32(3.345)$

$=$ −107.033 m/s

12) $v(\theta, t) = 9.8 \sin(\theta)\, t$

$v(90°, t) = 9.8 \sin 90\, t$

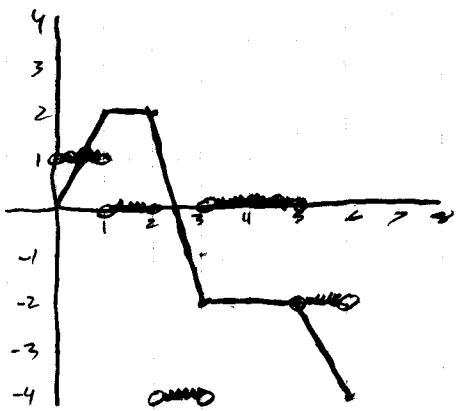
$= 9.8t$

b) $a(t) = 9.8$

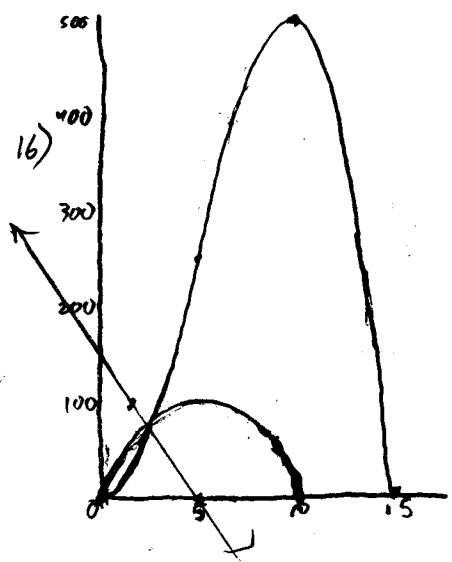
14) left $(2,3) \cup (5,6)$

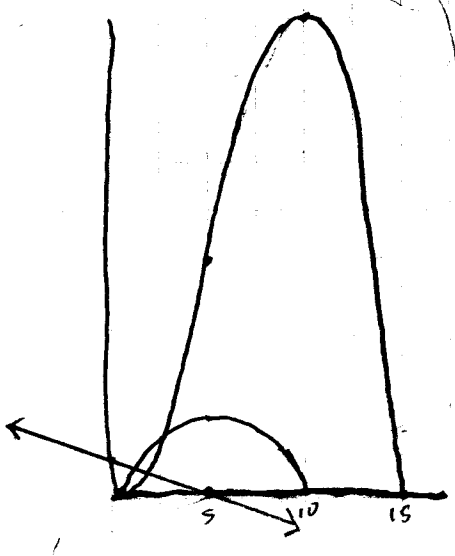
right $(0,1)$

stationary $(1,2) \cup (3,5)$



16)





$30t - 3t^2 = \frac{dt}{dy}$

$30 - 6t = \frac{d^2t}{dy}$

22) $b(t) = 10^6 + 10^4 t - 10^3 t^2$

$b'(t) = 10^4 - 2 \cdot 10^3 t$

$b(0) = 10^4$

$b(5) = 10^4 - 10 \cdot 10^3 = 0$

$b(10) = 10^4 - 2 \cdot 10^4 = -10^4$

24) $y(t) = 6\left(1 - \frac{t}{12}\right)^2 = 6\left(1 - \frac{t}{12}\right)\left(1 - \frac{t}{12}\right)$

$= 6\left(1 - \frac{t}{6} + \frac{t^2}{144}\right)$

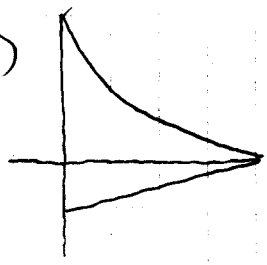
$= 6 - t + \frac{1}{24} t^2$

a) $\frac{dy}{dt} = \frac{1}{12} t - 1$

b) Fastest at $t = 0$, $= -1$

slowest at $t = 12$, $= 0$

c)



25) $V = \left(\frac{4}{3}\right)\pi r^3$

a) $\frac{dV}{dr} = 4\pi r^2 \Big|_2$ $4\pi(2)^2 = 16\pi$

b) $4\pi(2.2)^2 = 19.36\pi$

3.36π